Tutorial 10.

Preliminary:

① Dollar-weighted return:

A: balance in a fund at the start of the year;

B: balance in the fund at the end of the year;

C_k: net deposit at time t_k.

$A(1+i)+\sum_{k=1}^{n} C_k(1+i)^{1-t_k}$ is accumulate value. ~~i ...~~ $(1+i)^{(1-t)} \approx 1+(1-t)i$.

$$A(1+i)+C_1(1+(1-t_1)i)+\cdots+C_n(1+(1-t_n)i)=B$$

$$\left(A+\sum_{k=1}^{n} C_k\right)+i\left(A+\sum_{k=1}^{n} C_k(1-t_k)\right)=B$$

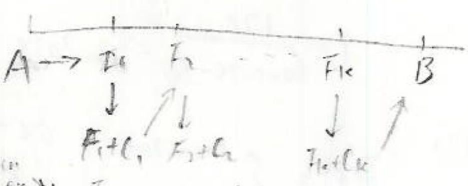
$$i=\frac{B-\left(A+\sum_{k=1}^{n} C_k\right)}{A+\sum_{k=1}^{n} C_k(1-t_k)}.$$

$I=B-\left(A+\sum_{k=1}^{n} C_k\right)$ is the net amount of interest.

② Time-weighted return:

F_k: the value of fund just before the net deposit C_k at the t_k.

$$i=\frac{F_1}{A}\times\frac{F_2}{F_1+C_1}\times\cdots\times\frac{F_k}{F_{k-1}+C_{k-1}}\times\frac{B}{F_k+C_k}-1$$



$\frac{F_j}{F_{j-1}+C_{j-1}}$ is growth factor from t_{j-1} to t_j.

return in time t_k $\Rightarrow$ $j_1=\frac{F_1}{A}-1$, $j_2=\frac{F_2}{F_1+C_1}-1$, $\cdots$ $j_{k+1}=\frac{B}{F_k+C_k}$,

$(1+j_1)\cdots(1+j_{k+1})=1+i$.

③ Yield measure on a fund

$$i \doteq \frac{2I}{2\int_{t_1}^{t_2} F(t)\,dt-I}=\frac{2I}{F(t_1)+F(t_2)-I}.$$

the amount in the fund at time t is $F(t)$

Exercise:

5.2.3.

	Value	withdrawal
Jan 1, 2005	100,000	
April 1, 2005	103,000	-8000
Jan 1, 2007	103,992	

x is annual rate

the balance at the end of 2005:

$100{,}000(1+x)-8000(1+\frac{3}{4}x)$

at the end of 2006

$(100{,}000(1+x)-8000(1+\frac{3}{4}x))(1+x)=103{,}992.$

$\Rightarrow x=0.0625$

5.2.6.

dollar weighted return for K,

$$100(1+i) - X(1+\tfrac{1}{2}i) + 2X(1+\tfrac{1}{4}i) = 125$$

$$\Rightarrow X = 125 - 100(1+i),$$

time weighted return for L,

$$\frac{125}{100} \times \frac{105.8}{125-X} = 1+i \quad \Rightarrow \quad X = 125 - \frac{132.25}{1+i}$$

then $$125 - \frac{132.25}{1+i} = 125 - 100(1+i) \quad \Rightarrow \quad i = 0.15.$$

5.3.1.

(a) $F(0) = 500, \quad F(1) = 500 + 100 - 40 + 60 = 620. \quad I = 60,$

yield rate $$i = \frac{2I}{F(0)+F(1) - I} = \frac{120}{500+620-60} = 0.1132$$

(b) (i) $F(0) = 500 + 100 = 600, \quad F(1) = 620, \quad I = 60,$

$$i = \frac{120}{600+620-60} = 0.1034.$$

(ii) $$F(t) = \begin{cases} 500+20t & 0 \le t \le \frac{1}{4} \\ 600+20t & \frac{1}{4} \le t \le 1. \end{cases} \Rightarrow \int_0^1 F(t)dt = \int_0^{\frac{1}{4}} F(t)dt + \int_{\frac{1}{4}}^1 F(t)dt = 585.$$

$$i = \frac{120}{2 \times 585 - 60} = 0.1081.$$

$$F(t) = \begin{cases} 500+20t & 0 \le t \le \frac{1}{2} \\ 600+20t & \frac{1}{2} \le t \le 1. \end{cases}$$

(iii) $$\int_0^1 F(t)dt = \int_0^{\frac{1}{2}} F(t)dt + \int_{\frac{1}{2}}^1 F(t)dt = 560 \quad \Rightarrow \quad i = \frac{120}{2 \times 560 - 60} = 0.1132$$

$$F(t) = \begin{cases} 500+20t & [0, \frac{3}{4}) \\ 600+20t & [\frac{3}{4}, 1] \end{cases}$$

(iv) $$\int_0^1 F(t)dt = \int_0^{\frac{3}{4}} F(t)dt + \int_{\frac{3}{4}}^1 F(t)dt = 535 \quad \Rightarrow \quad i = \frac{120}{2 \times 535 - 60} = 0.1188$$

$F(t) = 500 + 20t$

(v) $$\int_0^1 F(t)dt = \int_0^1 500 + 20t \, dt = 500 + 10t^2 \Big|_0^1 = 510 \quad \Rightarrow \quad i = \frac{120}{2 \times 510 - 60} = 0.125.$$

Problem set 9: 5.1.5, 5.2.1, 5.2.2, 5.2.4, 5.2.5, ~~[illegible]~~ Tutorial 5.2.3, 5.2.6, 5.3.1

5.1.5.

(a) $30{,}000 = \frac{14{,}000}{1+i} + \frac{12{,}000}{(1+i)^2} + \frac{6{,}000}{(1+i)^3} + \frac{4{,}000}{(1+i)^4} + \frac{2{,}000}{(1+i)^5} \Rightarrow i = 0.1203$

(b) MIRR: $L(1+j)^5 = KS_{\overline{5}|10\%}$

$3000(1+j)^5 = 14{,}000(1+0.1)^4 + 12{,}000(1+0.1)^3 + \cdots + 2{,}000 \Rightarrow j = 0.1081.$

(c) $NPV = \frac{14{,}000}{1.1} + \frac{12{,}000}{(1.1)^2} + \cdots + \frac{2{,}000}{(1.1)^5} - 30{,}000 = 1126$

(d) $14{,}000 + 12{,}000 + 6{,}000 > 30{,}000$ and $14{,}000 + 12{,}000 < 30{,}000$

during 3rd year.

(e) $\frac{14{,}000}{1.1} + \cdots + \frac{2{,}000}{(1.1)^5} > 30000, \quad \frac{14{,}000}{1.1} + \cdots + \frac{4000}{(1.1)^4} < 30{,}000$

during 5th year.

(f) $I = \frac{\text{present value of cash inflows}}{\text{present value of outflows}}$

$I = \frac{\frac{14{,}000}{1.1} + \cdots + \frac{2{,}000}{(1.1)^5}}{30{,}000} = 1.0375.$

5.2.1.

The time-weighted return rate is $\left[\frac{F_1}{A} \times \frac{F_2}{F_1+C_1} \times \cdots \times \frac{F_k}{F_{k-1}+C_{k-1}} \times \frac{B}{F_k+C_k}\right] - 1.$

$\sqrt{\frac{1{,}310{,}000 - 250{,}000}{1{,}000{,}000} \times \frac{1{,}265{,}000 + 150{,}000}{1{,}310{,}000} \times \frac{1{,}540{,}000 - 250{,}000}{1{,}265{,}000} \times \frac{1{,}420{,}000 + 150{,}000}{1{,}540{,}000}} - 1$

$= 0.0910.$

5.2.2.

$\frac{12}{10} \cdot \frac{X}{12+X} - 1 = 0 \Rightarrow X = 60.$

$10(1+Y) + X(1+\frac{1}{2}Y) = X \Rightarrow Y = -0.25.$

5.2-4.

Jan 1st	50	
March 15	40	20
June 1	80	80
June 30	157.50.	

6-month time-weighted return is

$$\frac{40}{50} \times \frac{80}{40+20} \times \frac{157.50}{80+80} = 1.05$$

then ~~annual~~ annual return is $\sqrt{1.05} - 1 = 0.1025$.

One-year time weighted yield is

$$\frac{40}{50} \times \frac{80}{60} \times \frac{175}{160} \times \frac{X}{250} = 1.1025.$$

$\Rightarrow X = 236.25.$

5.2-5.

Jan 1st	1
July 1st	0.8
Jan 1st	1.0

$\frac{0.8}{1} \times \frac{1.0}{0.8} - 1 = 0.$ time-weighted return is 0.

$$100{,}000\,(1+i) + 100{,}000\,(1+\tfrac{1}{2}i) = \underset{\text{July, 1st}}{100{,}000} \times \frac{1}{0.8} + \underset{\text{Jan 1st}}{100{,}000} \times 1$$

$= 225{,}000$

$\Rightarrow i = 0.1667.$